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TRAJECTORY ASSESSMENT AND THREAT PRIORITIZATION

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February 1980

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Final Report for the Period January 1978 to October 1979

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This study investigates techniques for threat assessment and threat prioritization. It encompasses design and evaluation of algorithms for extrapolation of missile trajectories based on information and estimates about the current missile state. The objective of the threat assessment function is prediction of the defender-threat engagement based on estimates of the threat position, velocity and acceleration. The objective of the threat prioritization function is to rank threats in order that they may be effectively countered.
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In this effort several methods of deterministic trajectory assessment were investigated. Extrapolation of the threat trajectory to the point and time of closest approach is performed in each of these methods. Probability of survival estimates are derived based on this information, and a baseline threat prioritization technique is developed.

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FOREWORD

This technical report was prepared by Harry M. Dobbins and Michael J. Noviskey of the Fire Control Technology Group, Fire Control Branch, Reconnaissance & Weapon Delivery Division under Project/Task/Work Unit number 7629/08/41. It is the final report for the period January 1978 to October 1979 and was submitted for publication in December 1979.

TABLE OF CONTENTS

SECTION		PAGE
I	INTRODUCTION	1
	1. Statement of the Problem	1
II	TECHNICAL STUDIES	5
	1. Comparison of Trajectory Assessment Methods	5
	a. Constant Velocity Assumption	5
	b. Constant Acceleration Assumption	8
	c. Decreasing Acceleration Assumption	9
	2. Comparison of Methods	10
	3. Probability of Survival When Given No Action	13
	4. Threat Prioritization	19
III	CONCLUSIONS	21
	APPENDIX: Figures Illustrating Nominal Test Cases Using the Algorithms Developed Using the Constant Velocity Assumption and Acceleration Assumptions	23
	REFERENCES	34

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LIST OF ILLUSTRATIONS

FIGURE		PAGE
1	A Typical Engagement Scenario	3
2	A Comparison of Constant Velocity, A Constant Normal Acceleration, and A Decreasing Normal Acceleration in Aircraft-to-Missile LOS Frame	6
3	A Comparison of Actual Constant A_n Missile Trajectory and Predicted Miss Distances With Constant Velocity Normal to LOS, a Constant A_n, and Decreasing A_n in Aircraft-to-Missile LOS Frame	12
4	Probability of Survival as a Function of Miss Distance for Various Lethality Ranges	14
5	Probability of Survival vs Predicted Miss Distance for Varied Standard Deviation of x_0	17
6	Calculation of Probability of Survival Dependence on Standard Deviation for Various Miss Distances	18
7	Nominal Trajectory No. 1, Head-On AC at 45° to Missile Heading	24
8	Nominal Trajectory No. 2, Increased Initial Heading	25
9	Time-to-Go for Trajectory No. 1	26
10	Time-to-Go for Trajectory No. 2	27
11	Miss Distance for Trajectory No. 1	28
12	Miss Distance for Trajectory No. 2	29
13	Probability of Survival for Trajectory No. 1	30
14	Probability of Survival for Trajectory No. 2	31
15	Priority Ranking for Trajectory No. 1	32
16	Priority Ranking for Trajectory No. 2	33

SECTION I

INTRODUCTION

1. STATEMENT OF THE PROBLEM

In an airborne engagement in which a defending platform is subject to multiple simultaneous attack, decisions must be made concerning which threats must be countered and the order in which they must be countered. These decisions can be based on the time of arrival of the individual threats and the probability of survival of the defending platform based on extrapolation of the current threat trajectories.

It is important to note that the errors associated with extrapolation of threat trajectories are a function of two major error sources.

Measurement uncertainties, and

Model inaccuracies

Given, measurements and estimates of a threats position, velocity, and acceleration, estimates of the future threat state can be made by selecting a suitably sophisticated trajectory model. The problem lies in selecting a model which is useful in the sense that it is both computationally efficient and reasonably representative of the process involved. The effective use of this extrapolation approach requires evaluation of the error inherent in the model. In using this approach the estimate of the error must be included in the algorithm itself or decisions must be made depending on the specific scenario and knowledge of the algorithm's limitations.

Estimation of error places additional computational burden on the system, but provides a more generic algorithm in the sense that all scenarios may be treated by the same algorithm. The second approach requires *a priori* knowledge of error effects for classes of scenarios, the algorithms used, and the usually detailed identification information.

Figure 1 illustrates a typical engagement. The predicted miss distances depend on the assumptions made about the trajectory and noise in measurements. The constant velocity assumption (dashed straight line in the figure from missile (1)) indicates the trajectory when the missile had found an intercept course and is flying at essentially constant velocity. The errors inherent in the assumptions would not provide accurate miss distance estimates for missile (1) in the figure which is not on an intercept course but is still correcting to an intercept course.

The constant acceleration assumption (curved line in the figure from missile (1)) provides an improved estimate of miss distance. But now noise in the acceleration estimates and actual acceleration due to differences from a constant velocity course would degrade the prediction for missile (2).

Finally, for missile (3) both assumptions would fail, since the threat is not on an intercept course but is executing a high g maneuver to correct. To assume a constant velocity fails to take into account the high normal acceleration. However, to assume that the acceleration will remain constant throughout the entire trajectory produces an overcorrection. The assumptions, error sources, and possible solutions to these problems will be discussed more fully in Sections 2 and 3.

Since the prediction of miss distance is sensitive to both the measurements and inherent assumptions about the threat trajectory, a highly sophisticated threat model would not solve the problem. Such a model would not only increase the computational burden of the system but would also be subject to its own set of <u>a priori</u> assumptions and measurement noise.

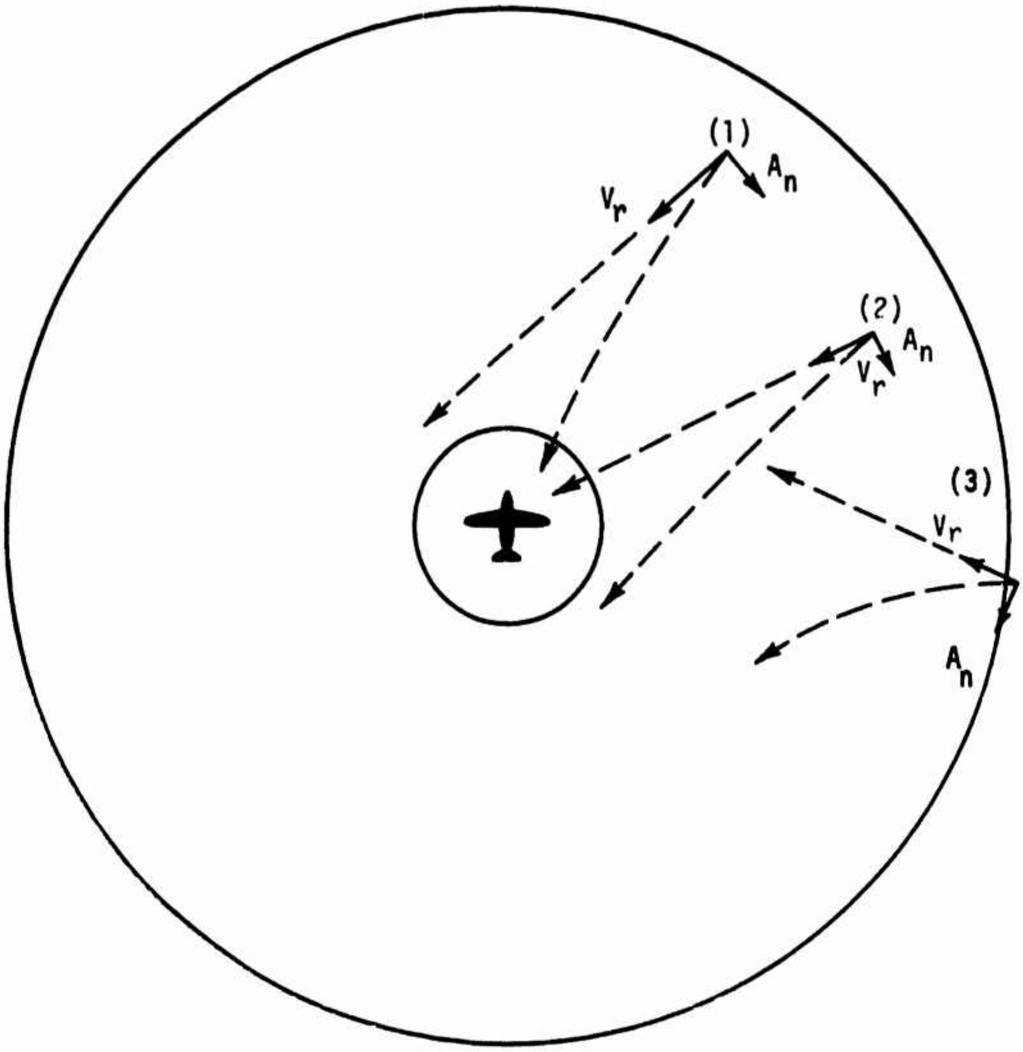

Figure 1. A Typical Engagement Scenario

An alternative solution would be to provide an estimate of the standard deviation for a number of successive predictions. If the threat trajectory is accurately determined by the model, the differences in prediction will be relatively small, and so will the standard deviation.

If the acceleration states are rapidly changing or the time-to-go value is large the standard deviation will be correspondingly large.

SECTION II

TECHNICAL STUDIES

1. COMPARISON OF TRAJECTORY ASSESSMENT METHODS

The purpose of the trajectory assessment function is to evaluate and rank targets with respect to the dangers they present to the defending platform, and the expected time of intercept or closest approach. These functions are based upon the state estimates of position, and the velocity and acceleration estimates provided by the sensors and tracking filters which can be used to predict or model its behavior.

a. Constant Velocity Assumption

Time-to-go and predicted miss distance are based on the assumption that both the defending platform and the threat missile maintain a constant relative velocity vector. The predicted miss distance is, then, the distance at the point of closest approach and the time-to-go is the time of closest approach. This calculation is somewhat simplified by adapting an intercept-plane coordinate frame. This plane is defined by the relative position and velocity vectors of the defender-threat pair. The aircraft and missile velocities normal to the intercept are equal, by definition. Furthermore, if all significant accelerations occur in the intercept plane the orientation of the intercept plane will not change as the scenario unfolds.

Figure 2 illustrates the geometric relationships involved, when it is assumed that the defending platform and threat maintain a constant velocity. The velocity of the threat relative to the platform is

$$V_t = (\dot{r}^2 + V_n^2)^{1/2} \tag{1}$$

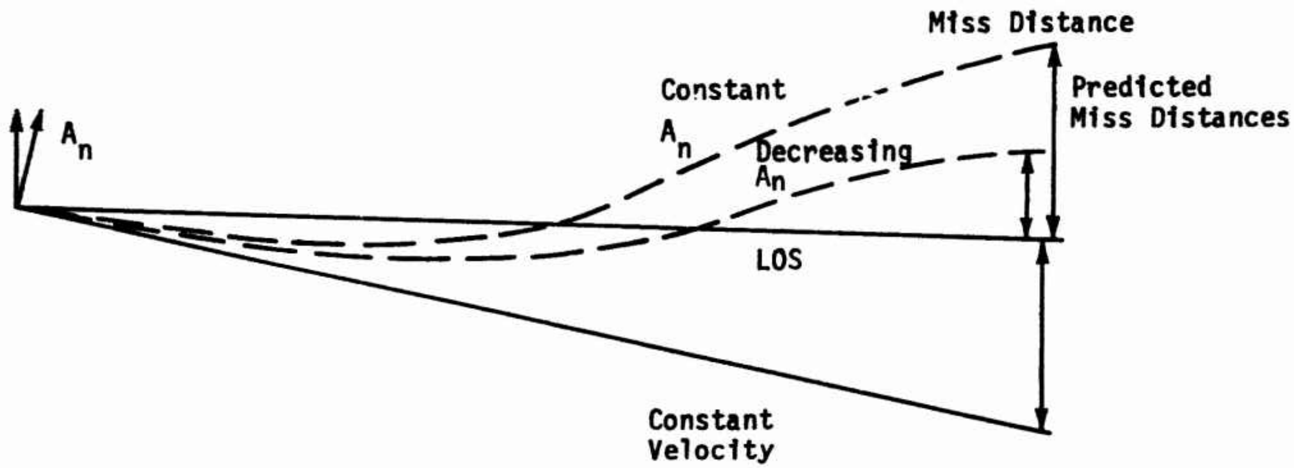

Figure 2. A Comparison of Constant Velocity, a Constant Normal Acceleration, and a Decreasing Normal Acceleration in Aircraft-to-Missile LOS Frame

where $\dot{r}$ is the velocity along the line of sight (LOS) and V_n is the velocity normal to the line of sight and is given by

$$V_n = r(\dot{\phi}^2 + \dot{\theta} \cos^2 \phi)^{1/2} \tag{2}$$

The heading error (α) is defined to be the angle of the LOS and the relative velocity vector as

$$\sin \alpha = V_n/V_t \tag{3}$$

From the geometry of the figure,

$$\sin \alpha = X_o/r \tag{4}$$

where X_o is the predicted miss distance, or point of closest approach, and r is the range. Then n can be calculated from

$$X_o = r\ V_n/V_t \tag{5}$$

and the time-to-go is, then,

$$T_{go} = -r\ \dot{r}/V_r^2 \tag{6}$$

In practice, however, a guiding threat does not maintain a constant velocity relative to the LOS. Small deviations from the true collision course would induce changes in V and $\dot{r}$, due to the missile guidance, even if the total velocity did remain constant. Since the total velocity actually changes due to boost and drag, the intercept course does change.

Assuming these changes in velocity are relatively small and the greatest source of error will be the measurement or estimate error, then, from Equation 5,

$$\frac{\Delta X_o}{X_o} = \frac{\Delta V_n}{V_n} + \frac{\Delta r}{r} - \frac{\Delta V_t}{V_t} \tag{7}$$

and, from Equation 6,

$$\frac{\Delta T_{go}}{T_{go}} = \frac{\Delta r}{r} + \frac{\Delta \dot{r}}{\dot{r}} - \frac{2\Delta V_r}{V_r} \tag{8}$$

Then, for a small heading error α, V_r will be large, compared to αV_t and r, and $\dot{r}$ will be large compared to ar and $a\dot{r}$. However, the normal velocity V_n will be of the same order of magnitude as αV_n. By definition of constant velocity intercept trajectory, $V_n = 0$. Small errors in the estimates of V_n will dramatically affect the miss distance prediction.

b. Constant Acceleration Assumption

The approach taken and discussed by the Charles Stark Draper Laboratories (Reference 2) includes the use of constant velocity and assumptions of both small axial and normal acceleration. Then the miss distance and time-to-go are expressed as

$$X_o = \vec{V}_o T_{go} + \tfrac{1}{2} A_o T^2_{go} \tag{9}$$

and

$$T_{go} = \frac{(\vec{r}_o \cdot \vec{V}_o)}{|V_o^{\,2}|} \tag{10}$$

where

$\dot{r}_o$ is the missile initial position

V_o is the initial velocity

A_o is the initial and assumed constant acceleration

Equations 9 and 10 will provide exact results in the event that a coasting missile approaches the defending platform along its implied constant velocity vector. Any deviation from this set of circumstances would result in an undetermined error associated with the estimator. Reference 2 details a more complicated version of Equation 9 which would provide an enhanced computation of multi-aspect threat arrivals, but drag and staging will still result in accelerations not modeled by the estimator (Reference 3). This approach also suffers because it does not attempt to model the missile guidance. The normal acceleration will not remain constant, due to guidance, but will tend toward zero as the intercept course is attained.

c. Decreasing Acceleration Assumption

To account for decreasing normal acceleration as the intercept course is attained the following assumptions are made. The normal acceleration decreases linearly with time and at T_{go} the normal velocity reaches zero. Then the normal velocity becomes

$$V_n = \int A_n(1 - T/T_{go})d_t \qquad (11)$$

$$V_n = A_nT - \tfrac{1}{2}A_n T^2/T_{go} + V_o \qquad (12)$$

and the miss distance becomes

$$X_o = \int_{V_o}^{V_n} V_n \; d_t \qquad (13)$$

$$= \int_{0}^{T_{go}} \left(A_n T - \tfrac{1}{2} A_n T^2 / T_{go} + V_o\right) d_t \qquad (14)$$

$$= \tfrac{1}{2} A_n T^2 - 1/6\left(A_n T^3 / T_{go}\right) + V_{ot} \Big|_{0}^{T_{go}} \qquad (15)$$

At T_{go}

$$X_o = 1/3\left(A_n T_{go}\right) + V_o T_{go} \qquad (16)$$

Then the time-to-go can be calculated from the radial acceleration and velocities (assumed constant) by

$$T_{go} = \frac{V_o + \sqrt{V_o^2 + 2A_r r_o}}{A_r} \qquad (17)$$

Although this method attempts to account for missile guidance, no specific guidance is modeled, and the effects of staging and nonlinear drag effects are still not modeled.

2. COMPARISON OF METHODS

All of the methods discussed are based on a polynomial approximation of the threat trajectory and extrapolation to the point of closest approach. The constant-velocity method is based on linear or straight-line approximation, and does not account for acceleration. The constant-acceleration method uses a quadratic approximation, but does not account for guidance. The decreasing acceleration method is based on a cubic approximation, which is reduced to a quadratic expression by making assumptions about the threat trajectory.

The major problem associated with each of these methods is a divergence of the predicted miss distance from the actual miss distance for large time-to-go. Since the value of function of t and its derivities are specified, the error associated with this type of extrapolation has an error of the form (Reference 6).

$$\varepsilon = F(t) - P(t) \tag{18}$$

$$\varepsilon = F(t) - \frac{A_0 T^n}{n_1} - \frac{A_0 T^{n-1}}{(n-1)_1} + \ldots + A_n) \tag{19}$$

If P(t) is an accurate representation of F(t), the error grows in accordance with the power of the number of steps extrapolated forward multiplied by the error in the coefficients. Figure 2 illustrates the error associated with the methods discussed. As indicated, the constant velocity assumption generally gives poorer estimates than either a constant or a decreasing acceleration assumption. The constant acceleration assumption provides the best estimate when the threat must actually maintain constant acceleration to arrive at the point of closest approach, otherwise the decreasing acceleration assumption provides a better estimate. The major disadvantage to using these approaches is the lack of inherent measure of validity. That is, there is no measure of σt_{go} and σ (Reference 3). However, since the miss distance must be repetitively evaluated in real time, the average and standard deviations associated with the miss distance can also be calculated by

$$\sigma_{X_0} = \sum_{i=1}^{n} \frac{(X_{o_{av}} - X_{o_i})^2}{(n - 1)} \tag{20}$$

where X_{o_i} is the ith estimate of miss distance

$X_{o_{av}}$ is the average value of the last N estimates

N is the number of estimates

σ_{X_o} is the standard deviation over the last N estimates

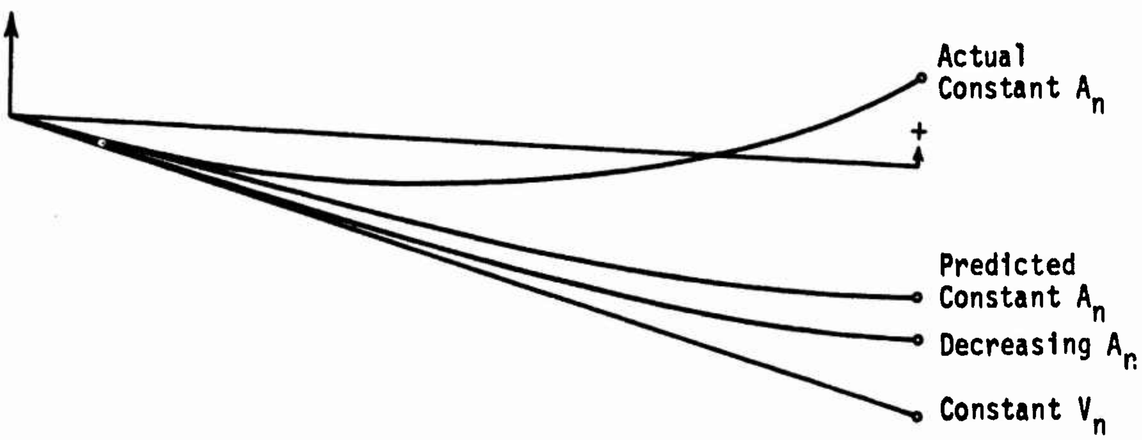

Figure 3. A Comparison of Actual Constant A_n Missile Trajectory and Predicted Miss Distances With Constant Velocity Normal to LOS, a Constant A_n, and Decreasing A_n in Aircraft-to-Missile LOS Frame

The standard deviation calculated, not only provides a measure of validity of the model used, but can also be used directly in the calculation of probability of survival.

3. PROBABILITY OF SURVIVAL WHEN GIVEN NO ACTION (PS/NA)

When given that a missile detonates at a distance x from an aircraft, we can assume the probability of survival depends on the detonation distance x and the warhead lethality range r_0 and express it as

$$PS/NA = 1 - e^{-x^2/r_0^2} \tag{21}$$

For simplicity, assume that r_0 is not a function of the angular position of the missile relative to the aircraft's velocity vector. Then the value of r_0 defines the 1 sigma radius for a given lethality. Without identification information the value of r_0 would be unknown. In this case r_0 would be selected as a maximum desirable keep-out-of-range.

A defensive system does not have absolute knowledge of what the value of x is before the detonation of the missile. The defensive system can make an estimate of the point of closest approach and can make an estimate of the time-to-go before the missile is at the point of closest approach. The error between the estimated point of closest approach and the actual point of detonation results because the defensive system cannot accurately predict the dynamic motion of the missile for long periods of time. In general, the higher the value of the standard deviation of x before the missile is at the point of closest approach, the more uncertain the estimate of the point of closest approach (x_0) will be.

Assume the probability density of a missile having a detonation distance of x to be:

$$\frac{dP(x)}{dx} = N e^{-\left(\frac{x-x_0}{\sigma x_0}\right)^2} \tag{22}$$

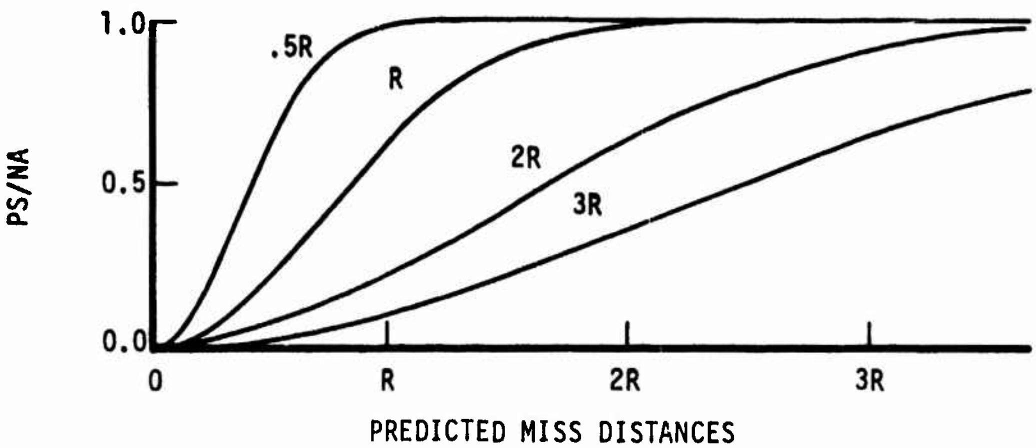

Figure 4. Probability of Survival as a Function of Miss Distance for Various Lethality Ranges

where σ_{x_0} is the standard deviation of the value of x_0, and N is the normalization coefficient. If $\sigma_{x_0} = 0$, then

$$\frac{dP(x)}{dx} = \delta(x-x_0) \tag{23}$$

The value of N can be determined because the total probability over all values of x from 0 to ∞ must be one

$$1 = N \int_0^\infty e^{-\left(\frac{x-x_0}{\sigma_{x_0}}\right)^2} dx \tag{24}$$

Let

$$\frac{x-x_0}{\sigma_{x_0}} = \frac{y}{2} \tag{25}$$

Then

$$dx = \frac{\sigma_{x_0}}{2} dy \tag{26}$$

and

$$\frac{1}{N} = \int_{-\frac{2x_0}{\sigma_{x_0}}}^{\frac{\sigma_{x_0}}{2}} \frac{\sigma_{x_0}}{2} e^{-y^2/2} dy \tag{27}$$

$$\frac{1}{N} = \frac{\sigma_{x_0}\,\pi}{2} \frac{2}{\pi} \int_0^\infty e^{-y^2/2} dy + \frac{2}{\pi} \frac{2x_0}{\sigma_{x_0}} e^{-\left(y^2/2\right)} dy \tag{28}$$

The error function is defined as

$$\mathrm{ERF}\left(\frac{t}{2}\right) = \frac{2}{\pi}\int_0^t e^{-y^2/2}\, dy \tag{29}$$

$$\mathrm{ERF}\,(\infty) = 1 \tag{30}$$

$$\frac{1}{N} = \frac{ct_{go}\;\pi}{2}\left\{1 + \mathrm{ERF}\left(\frac{x_o}{\sigma_{x_o}}\right)\right\} \tag{31}$$

$$N = \frac{2}{\sigma_{x_o}\;\pi\left\{1 + \mathrm{ERF}\left(\frac{x_o}{\sigma_{x_o}}\right)\right\}} \tag{32}$$

$$\frac{dP(x)}{dx} = \frac{2\, e^{-\left(\frac{x-x_o}{\sigma_{x_o}}\right)^2}}{\sigma_{x_o}\;\pi\left\{1 + \mathrm{ERF}\left(\frac{x_o}{\sigma_{x_o}}\right)\right\}} \tag{33}$$

The probability of survival (given that no action is taken) is, then, the integrated product of the probability density of the point of detonation occurring at x and the probability of surviving the detonation at x, and can be written

$$PS/NA = \int_0^\infty \frac{dP(x)}{dx}\left(1 - e^{-\left(x^2/r_o^2\right)}\right) dx \tag{34}$$

Thus,

$$PS/NA = \left\{1 - \frac{r_o e^{-\left(\frac{x_o^2}{r_o^2 + \sigma^2_{x_o}}\right)}}{\left(r_o^2 + \sigma_{x_o}^2\right)\left\{1 + \mathrm{ERF}\left(\frac{x_o}{ct_{go}}\right)\right\}}\right\}\left\{1 + \mathrm{ERF}\left(\frac{r_o x_o}{\sigma^2_{x_o}\;\; r_o^2\;\; \sigma^2_{x_o}}\right)\right\} \tag{35}$$

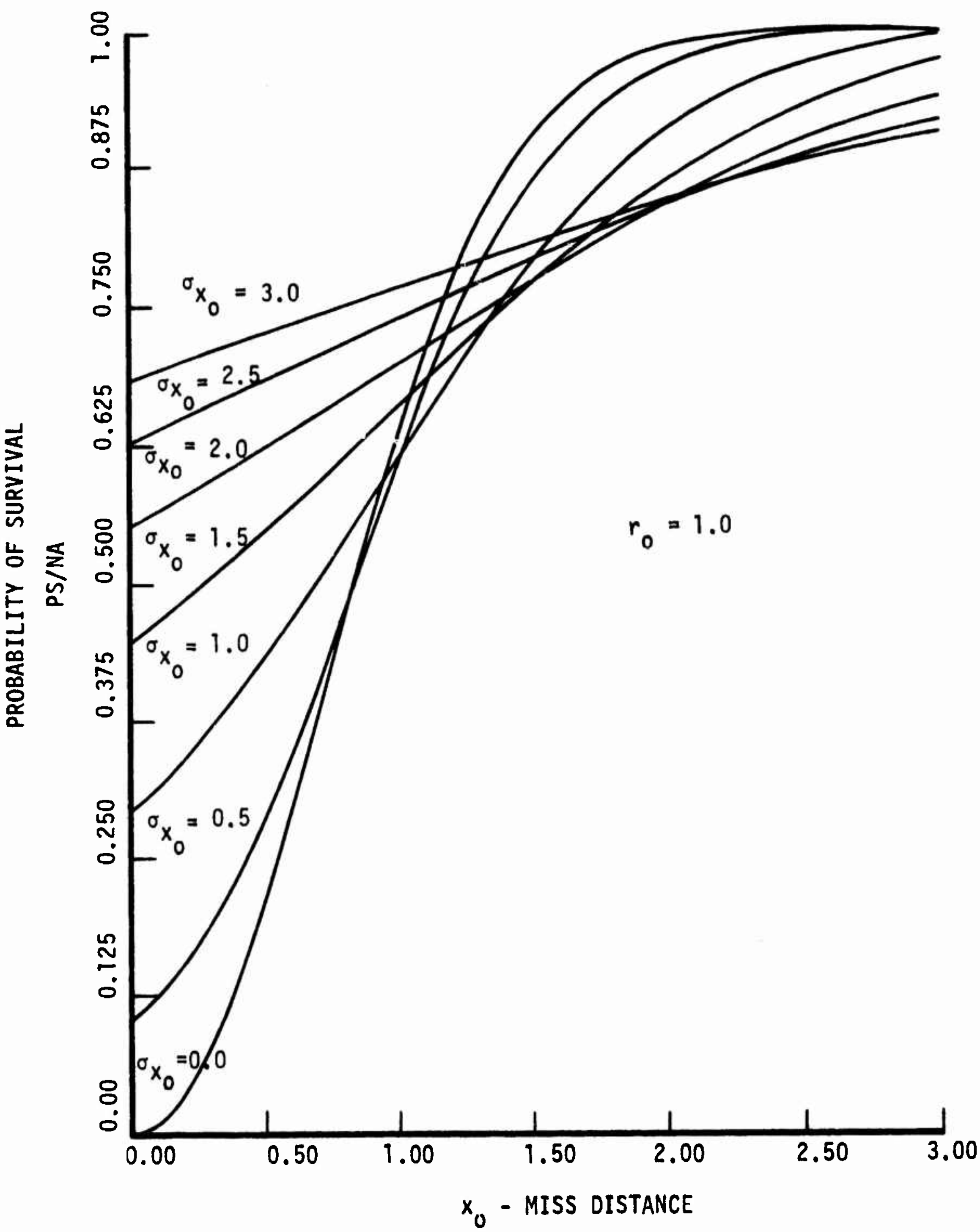

Figure 5. Probability of Survival vs Predicted Miss Distance for Varied Standard Deviation of x_0

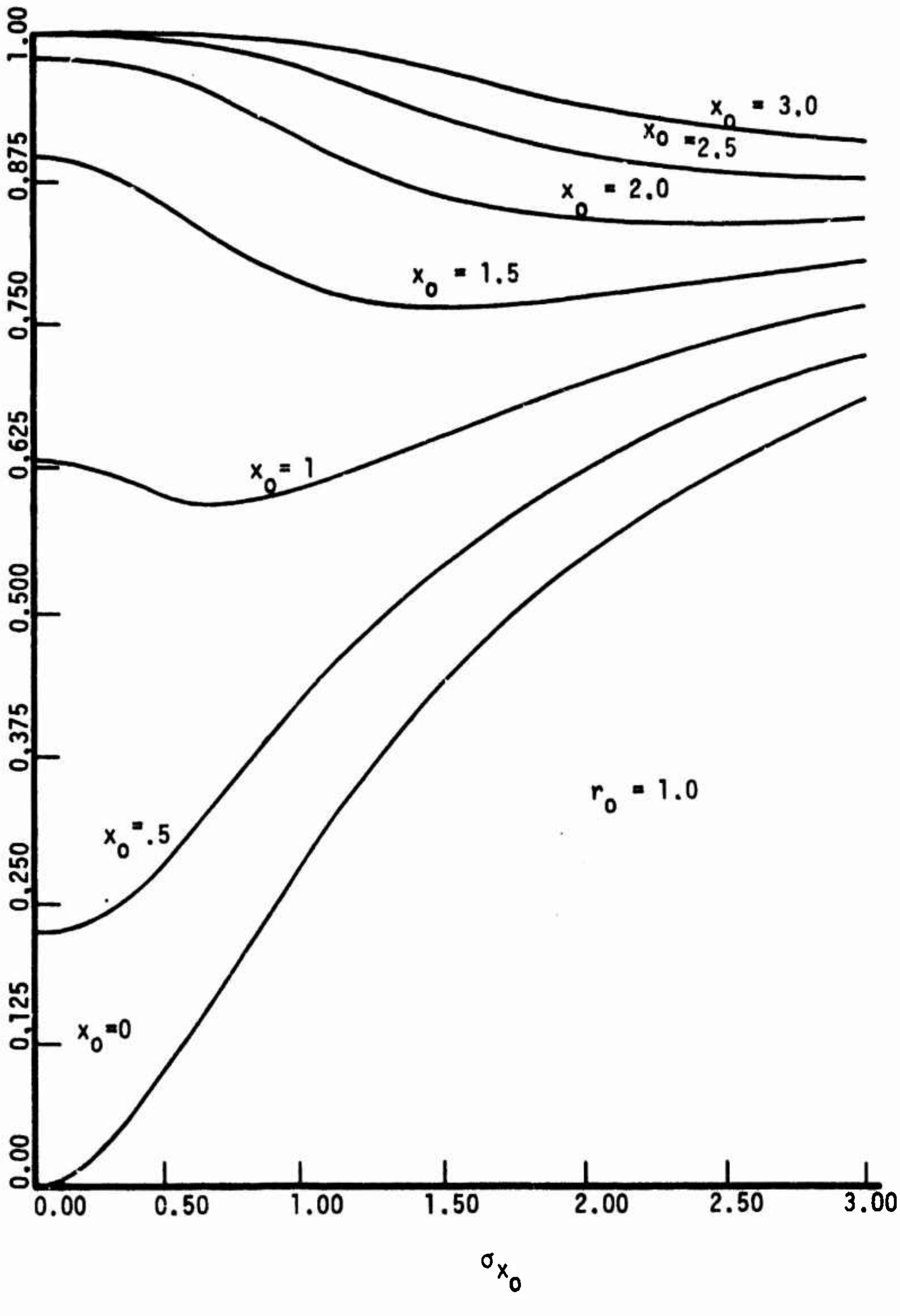

Figure 6. Calculation of Probability of Survival Dependence on Standard Deviation for Various Miss Distances

Let x_0 and σ_{x_0} be expressed in units of r_0, so that

$$PS/NA = \left\{1 - \frac{e^{-\left(\frac{x_0^2}{1+\sigma^2_{x_0}}\right)}}{\left(1 + \sigma^2_{x_0}\right)\left[1 + \left(ERF\ \frac{x_0}{\sigma_{x_0}}\right)\right]}\right\}\left\{1 + ERF\left(\frac{x_0}{ct_{go}\ 1 + \sigma_{x_0}}\right)\right\} \quad (36)$$

Figures 5 and 6 illustrate the effect of uncertainty or model error for various miss distances. Figure 5 shows that as estimated miss distance increases the probability of survival also increases. Figure 6 shows that as the estimate improves (the modeled behavior of the threat resembles the actual behavior), better estimates of probability of survival are generated. As σ_{x_0} becomes very large compared to miss distance the estimated probability of survival becomes $PS/NA = 1-e^{-1}$.

Figure 7 gives a contour plot of the probability of survival for miss distance and σ_{x_0}. Note that as the predicted miss distance and σ_{x_0} increase, the probability of survival estimate is less sensitive to variations in miss distance.

4. THREAT PRIORITIZATION

The priority of the threat is based on the probability of survival and T_{act} and the time or range at which a defensive action can be taken.

The time at which defensive action can begin can be calculated from

$$T_{act} = (R - R_{act})/\tfrac{1}{2}$$

For the constant velocity assumption, where

R is the range,

R_{act} is the range at which action can begin, and

V_L is the velocity along the line of sight.

Or, in the constant acceleration case

$$T_{act} = \frac{-V}{A_L} + \sqrt{\frac{V_L^2 - 2(R - R_{act})A_L}{A_L}}$$

where

A_L is the acceleration along the LOS.

Then the priority can be expressed as

$$P = (1 - PS/NA)\ ERF\ ((T_{go} - T_{act})/T_{go})$$

The error function rapidly increases towards unity as T_{go} becomes less than T_{act}, in which case the priority is then determined by the probability of kill due to the threat.

SECTION III

CONCLUSION

The methods developed for deterministic extrapolation of threat trajectories to perform probability of survival and prioritization have several positive features:

- relative simplicity

- indication of relatively good accuracy over a range of test scenario (References 1 and 2).

The problems associated with use of these methods are error sources:

- measurement errors and estimation inaccuracies

- minor model inaccuracies (the missile guidance laws are not directly modeled) which cause divergence of the extrapolated trajectory from the actual trajectory.

- major model inaccuracies, such as staging and nonlinear drag effects.

The major problem associated with deterministic extrapolation has been the lack of statistical measurement of the validity of the model. The statistical information, standard deviation of miss distance, can be directly calculated from N sequential estimations and directly used to determine probability of survival by the method developed in Section II.3. This reduces the sensitivity of the probability of survival calculation when the trajectory model is inaccurate or is giving rapidly N changing results.

The model inaccuracies associated with missile guidance, staging and drag, would require detailed and complex models. This type of model would either require some type of identification information or methods of determining the missile's guidance gain in real time. There are

nonlinearities associated with staging and uncertainties in predicting when it would occur. Since the current techniques provide reasonable results, the additional complexities and increased computational burden of a more detailed model would seem to preclude the possibility of a real-time solution using a more complex model at this time.

The measurement errors and state estimation inaccuracies would be inherent in any system, but they become more critical when polynominal extrapolation techniques are used, since the error in estimation grows as the product of the error and a power of the number of time steps extrapolated forward.

Methods for reducing the effect of this error could include various data smoothing techniques, such as, the least-squares polynominal fit or the Chebyshev min-max polynominal fit. These techniques are attractive since the number of data points necessary to calculate the standard deviation required for the probability of survival estimation could also be used for data smoothing. This would reduce the more critical errors associated with the polynominal approximation.

Few data points are required since the degree of the polynominal approximation is low. For example, only four data points would be required to generate a Chebyshev min-max parabola, which is the highest degree polynominal discussed.

Although these techniques reduce the measurement error, they do add computational burden and do not eliminate model inaccuracies. Since model inaccuracies exist, repetitive calculations are necessary. Additional trade studies would be required to determine the benefit associated with increased accuracy versus increased computational burden for a particular system.

APPENDIX

Figures Illustrating Nominal Test Cases
Using the Algorithms Developed Using the
Constant Velocity Assumption and Acceleration
Assumptions

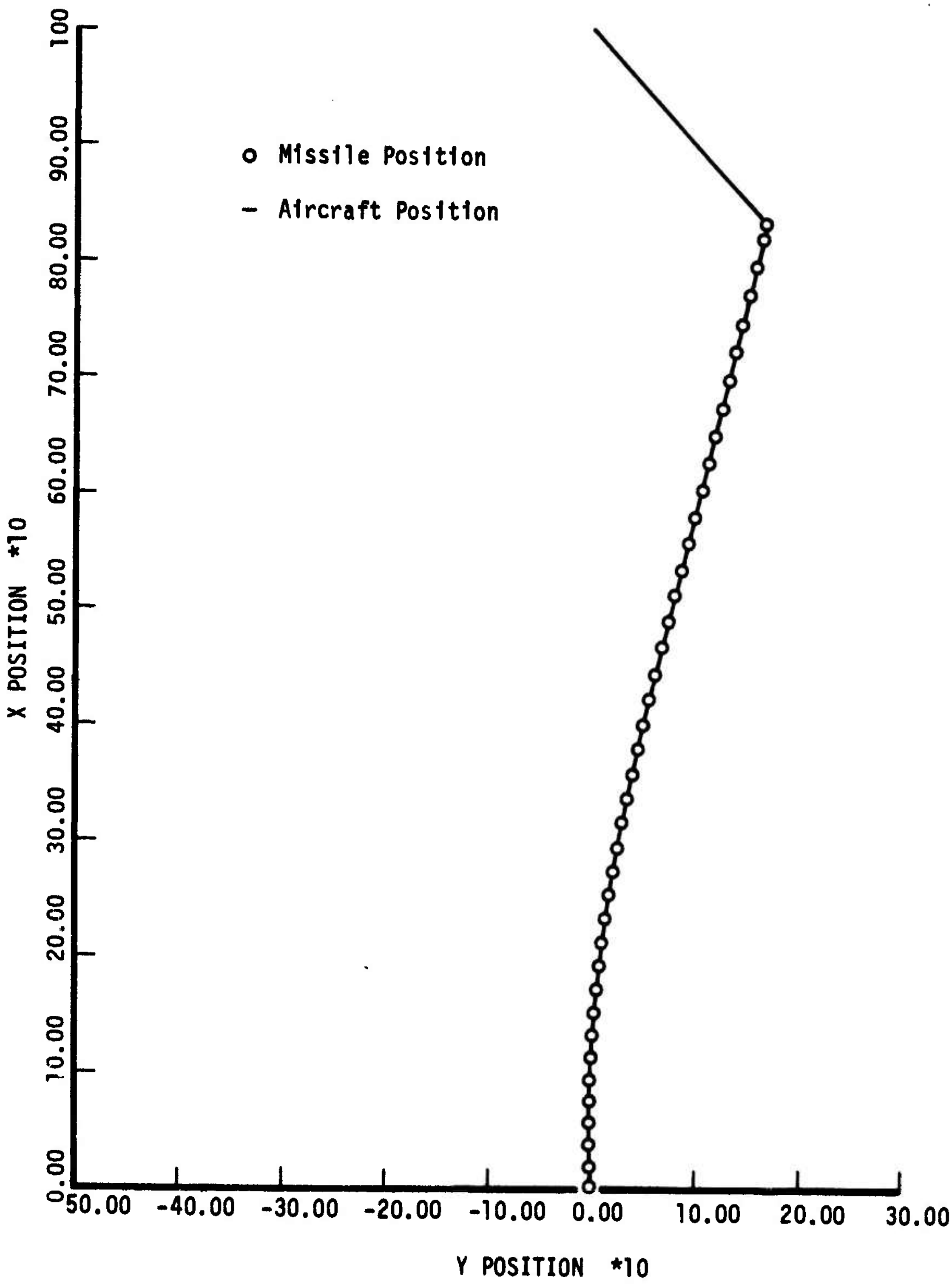

Figure 7. Nominal Trajectory No. 1, Head-On AC at 45° to Missile Heading

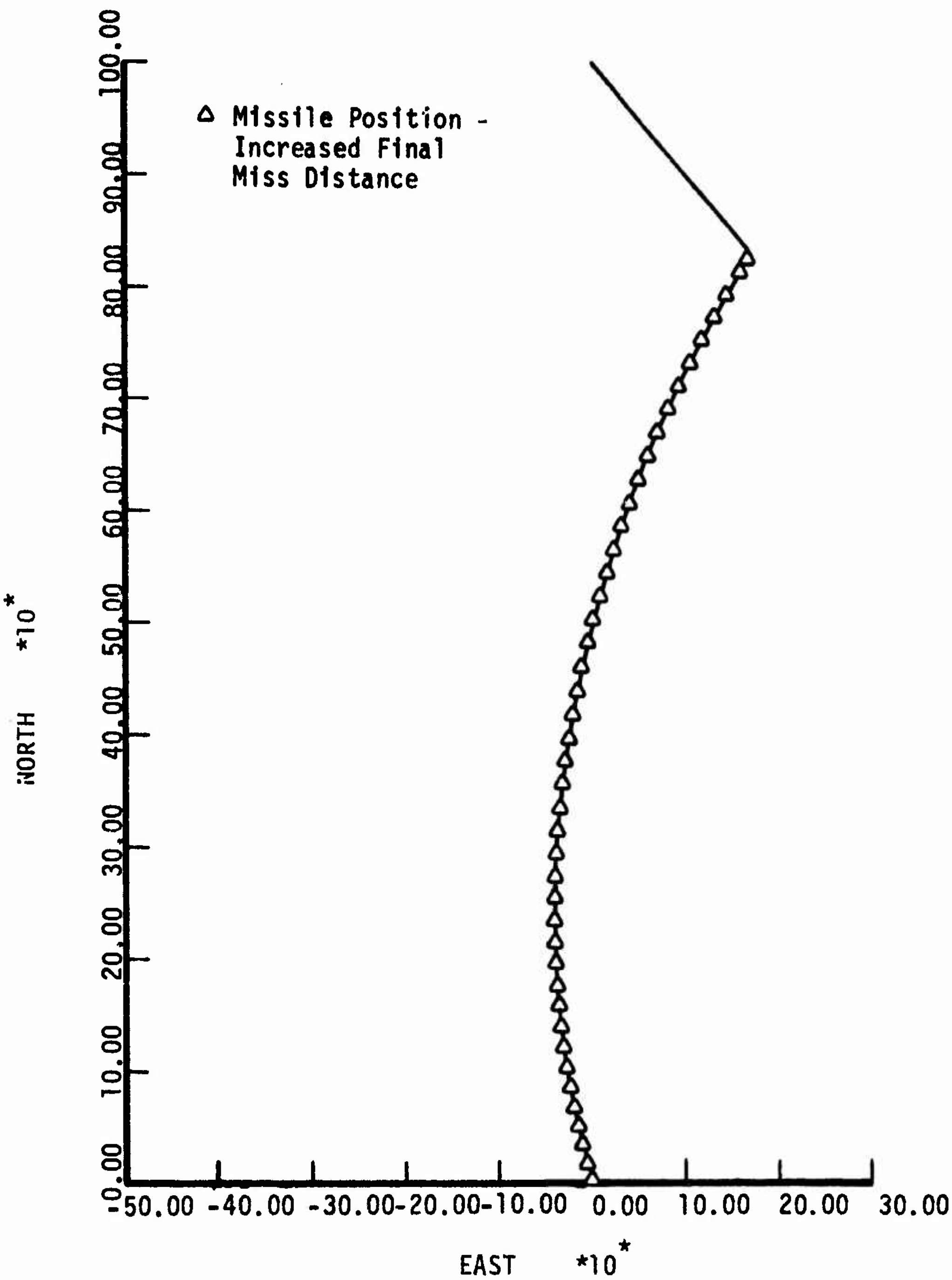

Figure 8. Nominal Trajectory No. 2, Increased Initial Heading

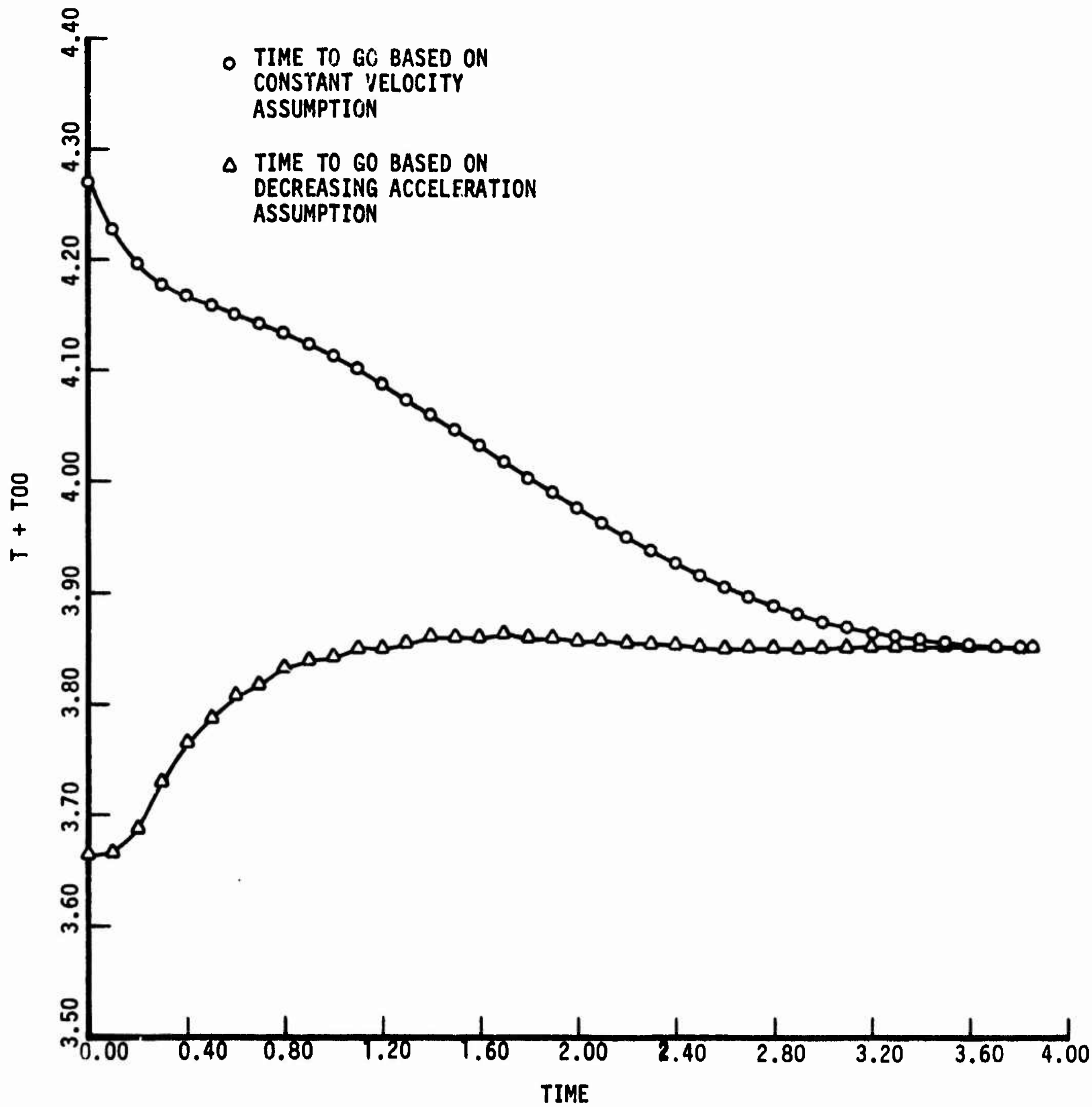

Figure 9. Time-to-Go for Trajectory No. 1

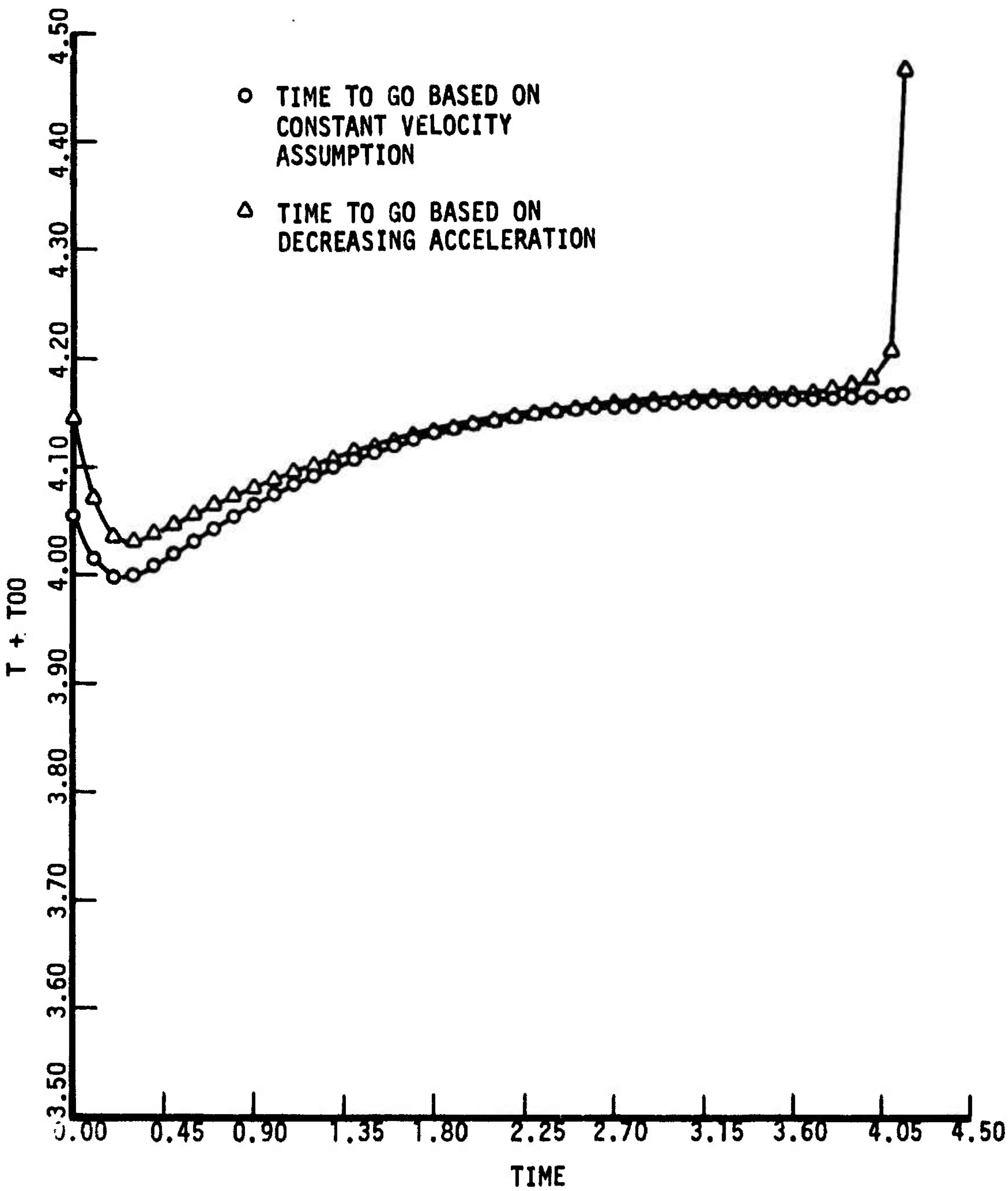

Figure 10. Time-to-Go for Trajectory No. 2

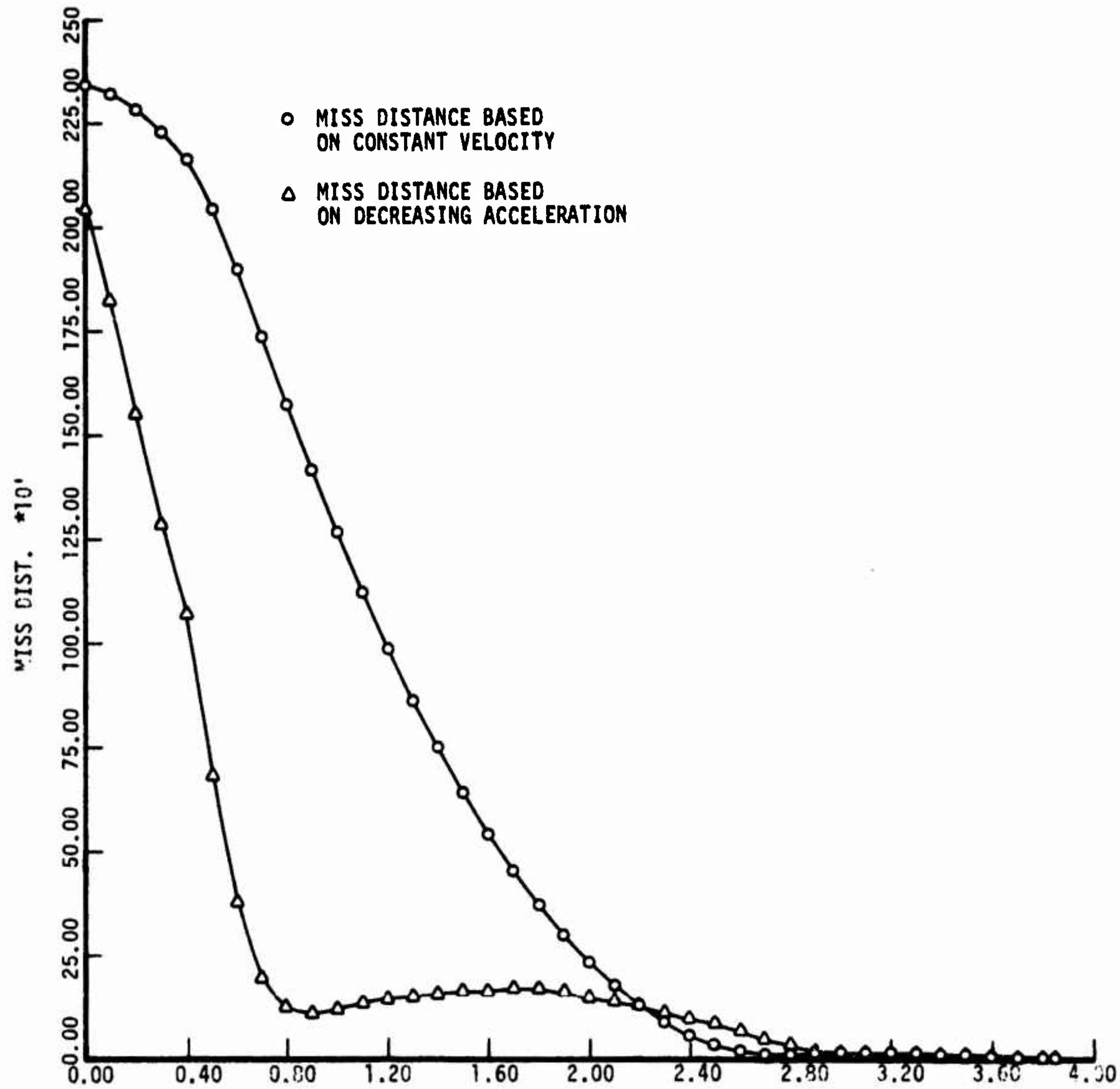

Figure 11. Miss Distance for Trajectory No. 1

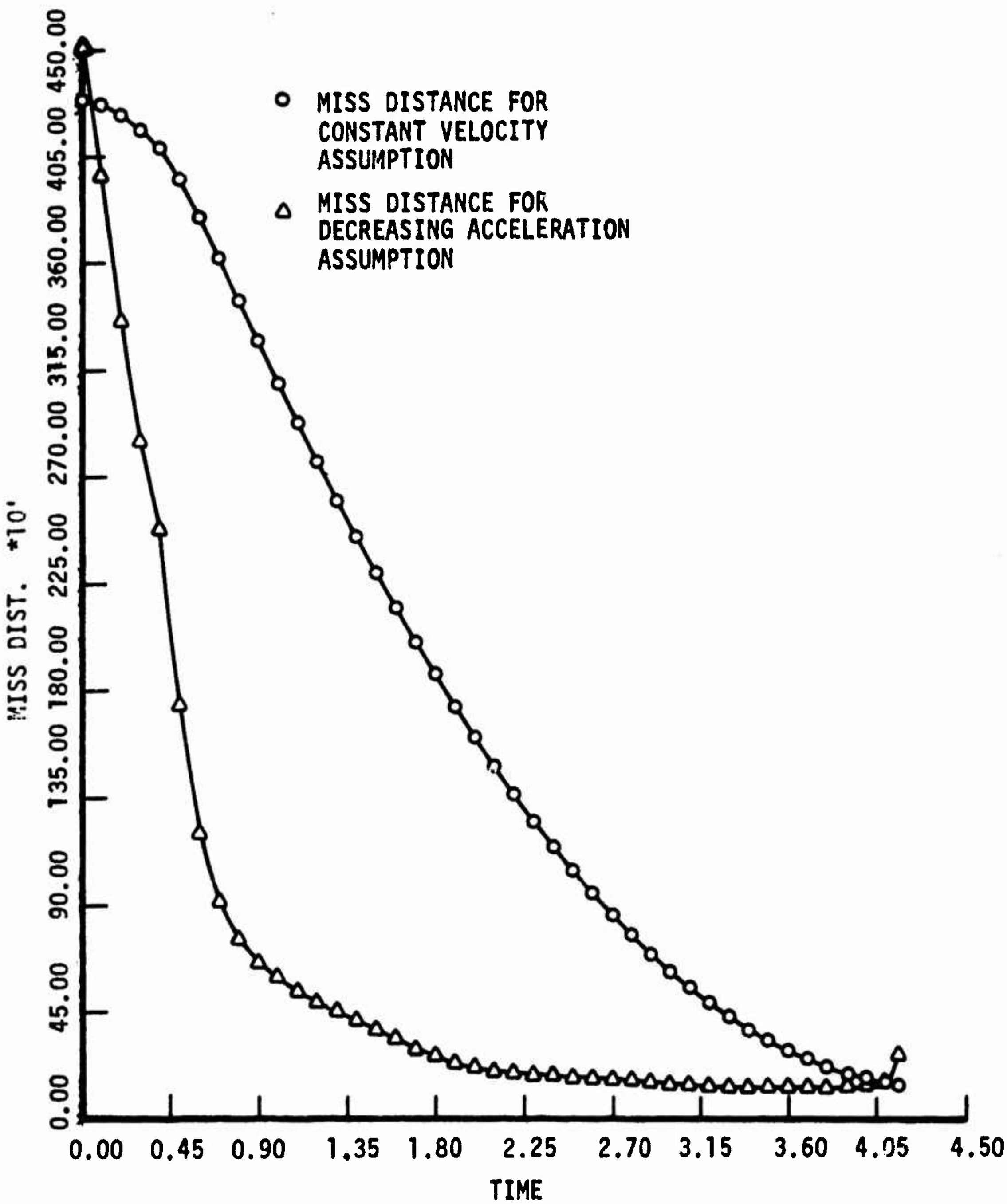

Figure 12. Miss Distance for Trajectory No. 2

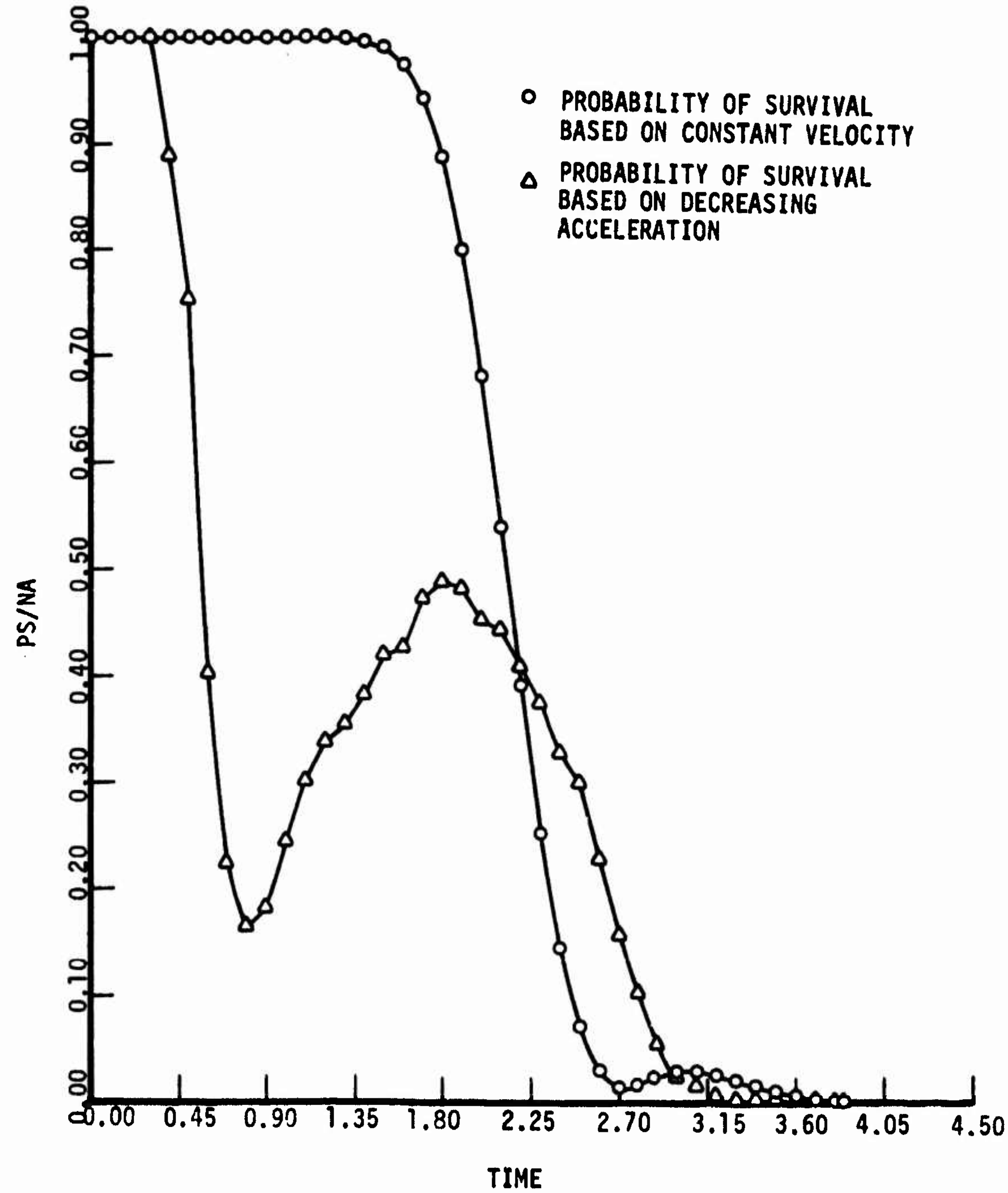

Figure 13. Probability of Survival for Trajectory No. 1

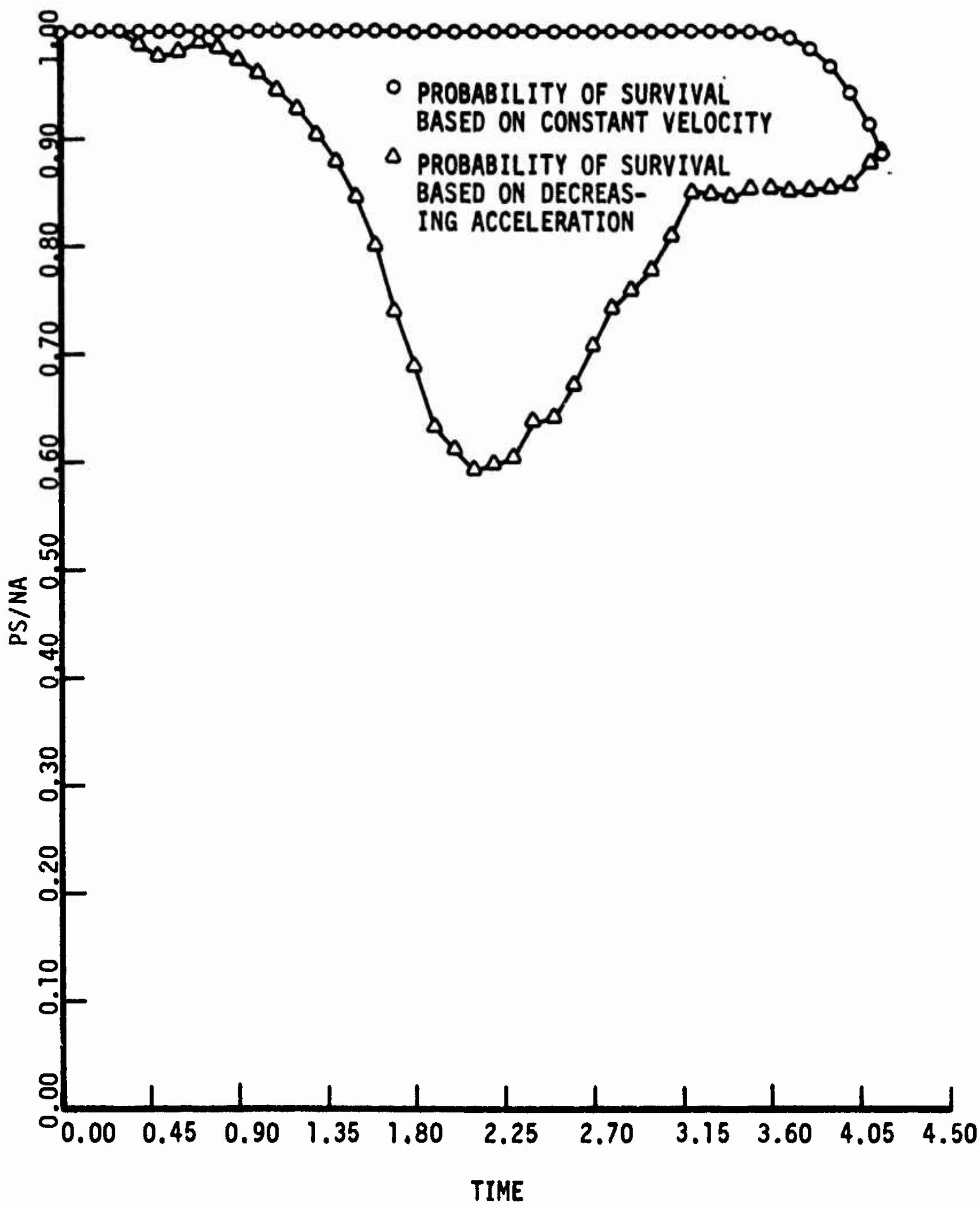

Figure 14. Probability of Survival for Trajectory No. 2

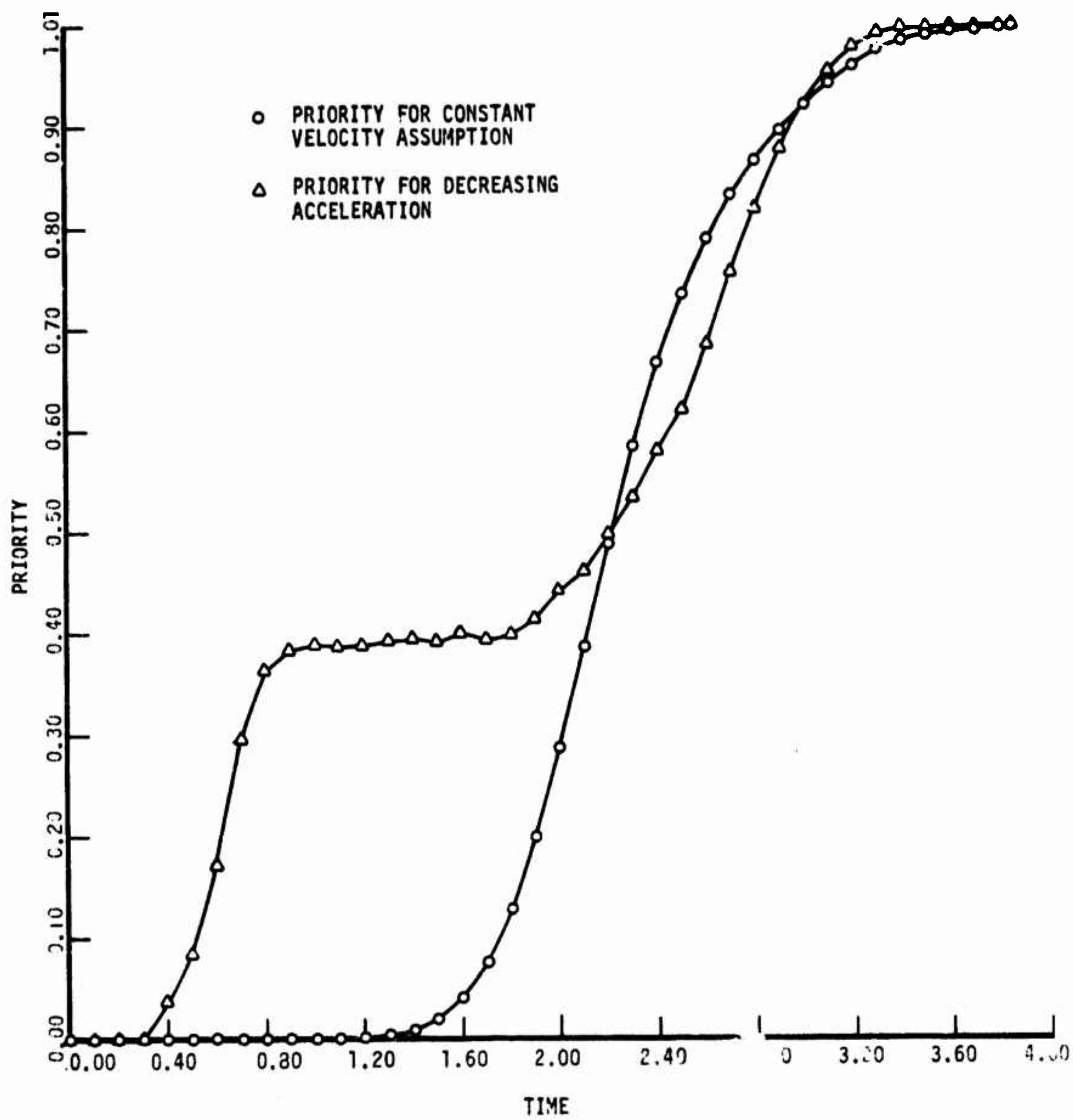

Figure 15. Priority Ranking for Trajectory No. 1

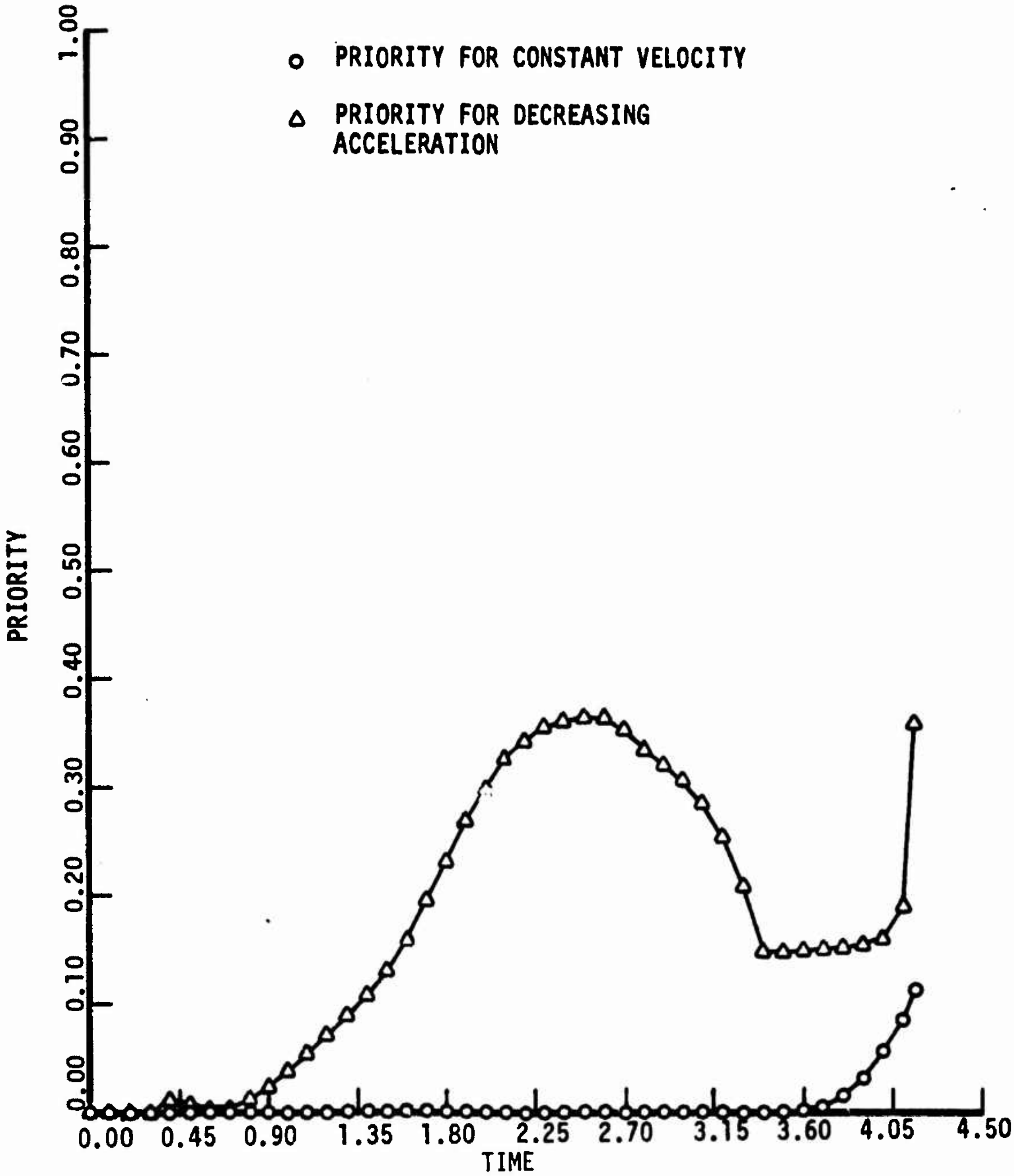

Figure 16. Priority Ranking for Trajectory No. 2

REFERENCES

1. Brown, C. M., et al, "Trajectory Estimation and Air-to-Air Collision Assessment;" presented at NAECON 77, May 1977.

2. Whitney, C. K., "Advanced Fire Control Algorithm Development for HEL Applications," AFAL-TR-78-147, Sep 1978.

3. Nash, J. M., "Fire Control Algorithm Mini-Task Report" (TERAC-Internal Report) TIR-002-78, 30 Jan 78.

4. Nash, J. M., "Simplified Algorithms for Airborne Fire Control/Sizing and Timing" (TERAC Internal) TIM-004-78, 13 Apr 78.

5. Brown, C. M., and Fiske, P. H., "Threat Assessment for Advanced Defensive Fire Control," AFAL TR-78-144, Sep 78.

6. Scheid, Francis, "Numerical Analysis," McGraw Hill Book Co., 1968.

7. Brown, C. M., DeAppolito, J. A., Fiske, P. H., and Price, C. F., "Advanced Defensive Fire Control Processing Methods Study," AFAL-TR-77-199, Oct 77.

8. Nash, J. M., "Advanced Defensive Fire Control Processing Methods Study," AFAL-TR-77-201, Sep 77.